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Notes

Application of Salzer's Inverse Laplace Transform Algorithm*

Salzer [1] has demonstrated that some inverse Laplace transforms could be obtained numerically by the use of

$$f(t) = (1/2\pi i) \int_{c-i\infty}^{c+i\infty} e^{pt} F(p)\, dp \cong \sum_{k=1}^{m} A_k^{(m)}(t)\, F(k). \qquad (1)$$

In this issue of the *Journal of Computational Physics*, Salzer [2] proposes a new algorithm; namely, replacing $p^s F(p)$ with an osculatory or hyperosculatory polynomial of variable $1/p$ and utilizing existing tables of Gaussian quadrature formulas [3, 4]. The new formula is

$$f(t) = (1/2\pi i) \int_{c-i\infty}^{c+i\infty} (e^{pt}/p^s)\{p^s F(p)\}\, dp \cong \sum_{i=1}^{n} A_i L_{2n-1}(t/p_i). \qquad (2)$$

Results generated via this new technique are compared with Salzer's earlier values [1]. Further examples given in Churchill [5] are also considered.

The problems taken from Salzer are

(a) $F(p) = \dfrac{1}{p+1}$,

(b) $F(p) = \dfrac{1}{(p^2+1)^{1/2}}$,

(c) $F(p) = \dfrac{1}{4}\,[-p^3 + 3a^2 p + (p^2+a^2)^{3/2}]^{1/2} \left[\dfrac{2\pi}{(p^2+a^2)^3}\right]^{1/2}$,

(d) $F(p) = \dfrac{e^{-x(p/k)^{1/2}}}{p-\alpha}$, where $k > 0$, $x > 0$, and α is unrestricted,

(e) $F(p) = \dfrac{e^{-x(p/k)^{1/2}}}{p^{3/4}}$, where $k > 0$, $x > 0$.

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The problems taken from Churchill are

(a) $F(p) = \dfrac{e^{-k/p}}{p^{1/2}}$,

(b) $F(p) = \dfrac{1}{p - \alpha}$,

(c) $F(p) = \dfrac{p}{p^2 + \alpha^2}$,

(d) $F(p) = \dfrac{\arctan(k/p)}{p}$,

(e) $F(p) = \dfrac{[(p + 2\alpha)^{1/2} - p^{1/2}]}{[(p + 2\alpha)^{1/2} + p^{1/2}]}$.

Initially, osculatory polynomials of order 3 and 8 and hyperosculatory polynomials of order 2 and 5 were selected for these test problems. In all cases, $s = 1$, so that tabular values in Stroud and Secrest [3] could be used. All problems were readily evaluated on a CDC 7600 computer in single or double precision using complex or strictly real arithmetic.

For Salzer's problem, with the exception of case (d), the results improve with increasing order *and* as hyperosculatory polynomials replace osculatory ones, as seen in Table I. Also, for Churchill's problem, as seen in Table II, the results for cases (c), (d), and (e) are excellent; however, case (b) for the hyperosculatory solution is unsatisfactory, and for case (a), neither solution is acceptable. Carrying out the calculation to higher order only improved the Churchill hyperosculatory

TABLE I

Salzer Problems

Order	Type	$a\ (t = 1.0)$	$b\ (t = 2.0)$	$c\ \begin{pmatrix} t = 1.6 \\ a = 0.2 \end{pmatrix}$	$d\ \begin{pmatrix} t = 2.4 \\ \alpha = 0.4 \\ k = 1.2 \\ x = 0.8 \end{pmatrix}$	$e\ \begin{pmatrix} t = 0.6 \\ k = 1.0 \\ x = 0.75 \end{pmatrix}$
	exact	0.367879	0.223890	0.397899	1.582262	0.557833
8	Salzer [1]	0.36791	0.2242	0.395993	1.59401	0.562963
3	osculatory	0.366379	0.232510	0.399154	1.550209	0.596681
8	osculatory	0.367879	0.223891	0.398092	1.259093	0.548802
2	hyperosculatory	0.365432	0.227065		1.536041	0.568495
5	hyperosculatory	0.367879	0.223890		1.593665	0.555509

TABLE II

Churchill Problems

Order	Type	$a\begin{pmatrix} t = 1.0 \\ k = 1.0 \end{pmatrix}$	$b\begin{pmatrix} t = 1.0 \\ \alpha = 1.2 \end{pmatrix}$	$c\begin{pmatrix} t = 1.0 \\ \alpha = 1.2 \end{pmatrix}$	$d\begin{pmatrix} t = 2.0 \\ k = 1.2 \end{pmatrix}$	$e\begin{pmatrix} t = 1.0 \\ \alpha = 1.0 \end{pmatrix}$
	exact	−0.234785	3.320116	0.362357	1.752485	0.207910
3	osculatory	−0.482295	−3.350000E1[a]	0.343449	1.752961	0.206473
8	osculatory	−0.170352	3.320301	0.362357	1.752489	0.207909
2	hyperosculatory	−0.676310	1.570000E2	0.339750	1.747151	0.205676
5	hyperosculatory	−0.414222	2.557062	0.362358	1.752485	0.207910
7	hyperosculatory	−1.508715	3.320055			
7D[b]	hyperosculatory	−1.510895[b]	3.320105[b]			

[a] $aE \pm n = a \times 10^{\pm n}$.
[b] Carried out in double precision.

case (b), as Table II shows in seventh order. Double precision calculations, also given in Table II, show that there is a large loss of significant numbers, but such losses cannot explain the remaining cases of discrepancies. Failure in Salzer case (d) and Churchill case (a) can be traced to the fact that the series expansion of $pF(p)$ is not well approximated by a polynomial in $1/p$.

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